

RESEARCH

Boolean algebra depends on two-element logic. C-algebra, Ada, A*- algebra and our Pre A*-algebra are regular extensions of Boolean logic to 3 truth-values, where the third truth-value stands for an undefined truth-value. The Pre A*- algebra structure is denoted by $(A, \wedge, \vee, (-)^\sim)$ where A is non-empty set, $\wedge, \vee$ are binary operations and $(-)^\sim$ is a unary operation.

The Doctoral thesis ALGEBRAIC STUDY OF CERTAIN CLASSES OF PRE A*-ALGEBRAS AND C-ALGEBRAS consists of **eight** chapters.

The Chapter-1 originate with the concept of Pre A*-algebra and some basic fundamental results of Boolean algebra. It also includes the useful properties of Pre A*-algebra. We discuss about the concept of centre of Pre A*-algebra and its useful characterizations. We mention the condition for which absorption law hold in a Pre A*-algebra and a result associated with Pre A*-homomorphism. We present the various methods of generation of Pre A*-algebras from Boolean algebra.

The Chapter-2 depicts the concept of *Semilattice structure on Pre A*-Algebras*, in this Chapter we define two binary operation $*$ and $\oplus$ on Pre A*-algebra A and such that $\langle A, * \rangle$ and $\langle A, \oplus \rangle$ become semilattices. We define $\leq_*$ and $\leq_\oplus$ on a Pre A*-algebra obtain the properties of Pre A*-algebra as a poset. We obtain modular type results with respect to these operations. We observe that element 2 acts as the least element with respect to $*$, where as the same element 2 acts as the greatest element with respect to $\oplus$ like the performance of a student in two different subjects, who obtains least mark in one subject and highest mark in the second subject. We give a number of equivalent conditions for Pre A*-algebra A to become a Boolean algebra in terms of the binary operations $*$ and $\oplus$ and the partial orders $\leq_*$ and $\leq_\oplus$. We derive the necessary conditions for $(A, \leq_*)$ and $(A, \leq_\oplus)$ are to become a lattice. We also obtain $x \leq_* y$ if and only if $y^\sim \leq_\oplus x^\sim$ and prove that the Pre A*-algebra becomes trivial if the partial orders $\leq_*$ and $\leq_\oplus$ are dual to each other. The main content of this chapter was published in Asian Journal of Scientific Research [15] and African Journal of Mathematics and Computer Science Research [14].

The Chapter-3 elucidates the concept of *Pre A*-Algebras with Order Relation*. Here we define a relation $\leq_c$ on A by $x \leq_c y$ iff $x \wedge y = x$ and $x \vee y = y$. We derive some important properties of $(A, \leq_c)$ which leads to number of equivalent conditions A to become a Boolean algebra in terms of this partial ordering. We establish necessary conditions for a poset to become a lattice. The main content of this chapter was published in International Journal of contemporary Mathematical Sciences [19].

The Chapter-4 explores the concept of *Pre A*- Modules and If-Then-Else Algebras Over Pre A*-Algebras*. We define the If-Then-Else operation $if_x(p, q) = (x \wedge p) \vee (x \sim \wedge q)$ on Pre A*-algebra A ($if_x(p, q)$ should be viewed as conditional “if x , then p , else q ”) and derive the most important properties of the operation $if_x(-, -)$ ($x \in A$). We observe that either x or p or q is 2 then $\Gamma_x(p, q)$ is 2. We establish the diagonal property on the If-Then-Else with a constraint, further we note that if the ternary operation defined on $B(A)$ then diagonal property holds good. Also we obtain the result that the variety **A-ITE** of all If-then-else algebras over Pre A*-algebras A and the variety **A-Mod** of Pre A*-modules over A are equivalent. The main content of this chapter was published in International Journal of Systemics, Cybernetics and Informatics [13].

The Chapter-5 *Prime and Maximal Ideals of Pre A*-Algebras* deals with the notation of an ideal, prime ideal, maximal ideal and minimal prime ideal of Pre A*-algebra A and discuss certain examples. We prove several fundamental properties of these, in particular we extend to prove that every ideal I of a Pre A*-algebra A is the intersection of all prime ideals of A containing I . Also, we prove that, for any ideals I and J of Pre A*-algebra A , $(I : J)$ is precisely equal to the intersection of all prime ideals containing J and not containing I . We also show that every maximal ideal is necessarily prime, while the converse is true for special cases only. The main content of this chapter was published in International Journal of Computational Cognition [17] and Trends in Applied Sciences Research [16].

In Chapter-6 *The Category of Pre A*-Algebras* we cram the Categorical aspects of Pre A*-Algebras. Here we establish the Categorical equivalence by showing that the category of **Pre A*** of Pre A*- Algebras is equivalent to the category **B** of Boolean algebras, to the category **A** of Adas and to the category **3-ring** of 3-rings. Also this chapter concerns with the study of Products

and Co-Products in the Category of Pre A^* -algebras. We obtain the results that the Product (Co-Product) of the family of Pre A^* -Algebras induces a Pre A^* -epimorphism (Pre A^* -monomorphism) algebras. The main content of this chapter was published in International Journal of Computational and Applied Mathematics [18] and Journal of Mathematical Sciences [21].

The Chapter-7 endow *congruences on Pre A^* -algebras*. We study two types of fundamental congruences on a Pre A^* -algebra A and discuss various properties of these. It is proved that for any element a of Pre A^* -algebra $\theta_a = \beta_a$. The identities of the quotient algebras A/θ_a and A/β_a were generated for both the operations $\wedge$ and $\vee$. We give sufficient conditions for two congruences on a Pre A^* -algebra A to be permutable. The main content of this chapter was published in International Journal of Applied Mathematics [22].

The Chapter-8 enlightens the concept of *Partial ordering on C-Algebra*. In this section we define a partial ordering $\leq$ on a C-Algebra by $x \leq y$ if $x \vee y = y$ and study the properties of C-Algebra as a poset. We derive necessary conditions for a C-algebra $\langle A, \wedge, \vee, ' \rangle$ to become a Boolean algebra. The main content of this chapter was published in International Journal of Computational Cognition [20].